INTERACTION BETWEEN A SHOCK WAVE AND A JET DRAINING INTO A SUPERSONIC COCURRENT FLOW WITH LESSER SUPERSONIC VELOCITY

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We wish to consider the impingement of a shock wave on a jet draining into a supersonic cocurrent flow. It is necessary to consider this class of flows in the course of solving different problems in internal and external aerogaseodynamics, for example, in calculating streams in undisturbed flows and jets. The influence of viscosity on the characteristics of the stream in front of the section at which the shock wave impinges on the jet is not considered, which is entirely justified in an initial segment of the jet when the width of the mixing zone is somewhat less than the initial width of the jet and the value of the turbulent viscosity in front of the interaction section is one-tenth the corresponding characteristic value in the zone of interaction with the shock wave.

In the present report we analyze the different forms of interaction between a shock wave and a jet, including regular interaction, Mach-type reflection in the jet, and totally sporadic reflection. It is shown that in the case of Mach interaction, it is impossible to calculate the position and dimensions of the Mach leg without also considering the influence of viscosity. An approximation method of calculating these types of flows that has been previously applied in calculating Mach interaction with a shock wave [1] is proposed.

1. It is assumed that the Mach numbers in a flow M_∞ and in a jet M_j satisfy the conditions $M_\infty > M_j > 1$. The initial width of the jet h_0 is taken to be the characteristic linear dimension, i.e., $h_0 = 1$. The shock wave impinging on the jet is characterized by the slope β_∞ relative to the direction of the current of the undisturbed flow. Different forms of interaction are possible depending on the values of the governing parameters of the problem, i.e., β_∞, M_∞, and M_j (Fig. 1).

A. Completely Regular Interaction. Upon interacting with the contact discontinuity surface A_1B_1, the impinging shock wave DA (Fig. 1a) divides into a refracted shock wave AO that propagates into the jet and a fan of rarefaction waves EAE' in the flow. The slope of the shock wave AO and of the rarefaction wave AE' and the values of the gas dynamic parameters for these waves are determined from the Rankine–Hugoniot and Prandtl–Mayer equations as functions of β_∞, M_∞, and M_j. In the case of regular reflection of the shock wave AO from the plane of symmetry of the flow OO_1, a reflected shock wave OB is formed whose slope and values of the gas dynamic parameters for this wave may be calculated from the formulas for an oblique shock, the well-known values of the parameters in the region AOB, and the condition of parallel flow in the region OBO'. Next, in interacting with the boundary of the jet A_1B_1 at the point B, the reflected shock wave OB divides into a refracted shock wave BC that propagates into the flow and a re-reflected shock wave BO, which propagates into the jet. It is no problem to calculate the slopes of these shock waves and the values of the associated gas dynamic parameters; this may be done by determining the breakdown of the discontinuity at the point B. Here we are using relationships that describe an oblique shock and a rarefaction wave, as well as the condition that the upstream and downstream pressure on the tangential discontinuity line BB', which is also a part of the boundary of the jet, are equal. The pattern of re-reflection of the shock wave BO' is repeated downstream until the stream in the jet and in the flow is again a parallel stream. At a sufficiently great distance from the zone of interaction downstream, the parameters of the shock wave BO' assume values that correspond approximately to regular reflection of the shock wave DA from the plane A_1A for a designated Mach number in undisturbed flow M_∞. The width of the flow is found from a condition of mass conservation in the jet. The pattern of the flow obtained as a result of the calculation and the corresponding totally regular interaction for $\beta_\infty = 22°$, $M_\infty = 5$, and $M_j = 3$ is depicted in Fig. 1a. Pressure is presented for each characteristic zone, adjusted to the corresponding value of the pressure in the undisturbed flow. The distinctive feature

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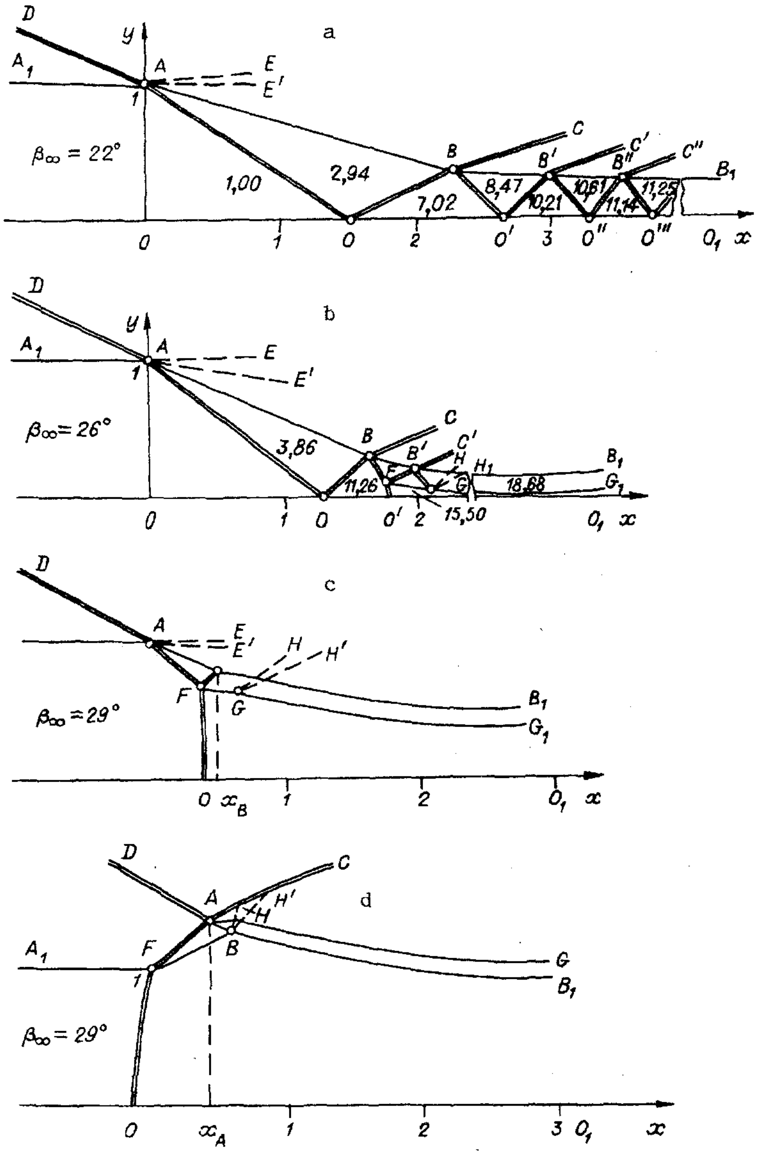

Fig. 1

of completely regular interaction is the fact that the gas flow remains supersonic as it passes through the system of shocks in the jet and the flow.

B. Mach Reflection in the Jet. If under the conditions of the flow we have just considered, the intensity of the refracted shock wave AO or of any of the re-reflected shock waves BO', B'O", etc., is such that regular reflection of any of these shock waves from the plane of symmetry OO_1 is impossible, Mach interaction will occur in the jet (Fig. 1b, 1c). For designated values of M_∞ and M_j, Mach interaction is observed once the angle of the incident shock β_∞ exceeds the critical value $\beta_{\infty cr}$ (M_∞, M_j). Under these conditions a powerful shock wave FO (or FO', FO",...) arises with subsonic flow behind and a weak reflected wave FB (or FB', FB",...). Once the reflected shock wave FB interacts with the contact discontinuity surface, a refracted shock wave BC and a re-reflected shock wave BG are formed. The weak shock wave BG interacts with the boundary of the subsonic region of the jet and is reflected from it. Since it is not possible for a pressure discontinuity to form in a neighborhood of the point G in the subsonic region of the jet, whereas such a discontinuity may occur in the transition through the re-reflected shock wave BG in the supersonic region of the jet, in order to restore the prior value of the pressure at the point G the shock wave B must be reflected from the boundary of the subsonic region by means of a fan of rarefaction waves HGH' at the same time that the boundary of the jet ruptures at this point. Moreover, the expansion waves HGH' are reflected from the shock wave BC (B'C',...). The variation of pressure along the subsonic region of the jet $O'FGG_1O_1$ is determined by the interaction conditions with the supersonic current in the jet in the region $FBB'B_1G_1G$. All the observations are also valid in the case of sporadic reflection of any of the re-reflected shock waves BO', B'O", etc.

C. Totally Sporadic Reflection. If the intensity of the incident shock wave DA is so great that the schemes of flow described in Part B cannot be realized, the flow is completely reconstructed (Fig. 1d). A slightly curved strong shock wave FO, nearly in the form of a straight line and turned convexly towards direction of the flow (unlike the analogous concave shock wave in the case of Mach-type reflection in a jet), behind which the flow is subsonic, is formed in the jet. The position of this wave is not known in advance and must be determined in the course of solving the problem. A weak shock wave FA forms in the free-stream flow whose slope, together with the slope of the strong shock wave FA at the point F, is determined from the Rankine–Hugoniot equations by proceeding on the basis of the conditions on the contact surface (equality of pressures and equality of slopes of the upstream and downstream velocity vectors). In the external flow the shock wave FA interacts with the incident shock wave DA at the point a, as a result of which two shock waves AC and AB together with a contact discontinuity surface AG form. The slopes of the shock waves AC and AB and the value of the gas dynamic parameters at the point A for these shock waves may be computed from the Rankine–Hugoniot equations and the conditions on the contact surface AG. As in the case described in Part B, upon interaction with the boundary of the jet at the point B, the shock wave AB is reflected from it by a fan of rarefaction waves HBH', and this is accompanied by a rupture in the shape of the boundary of the subsonic jet at the point B.

2. Let us consider a method of solving the problem. In parts B and C two schemes of interaction between an incident shock wave and a jet were described, both of which may be realized at the point of incidence within the framework of a model of inviscous flows. However, in attempting to calculate such flow schemes (i.e., determine the position and magnitude of the Mach leg) without going beyond the framework provided in this model, we end up at a contradiction. In fact, inasmuch as the slope of the contact surface at the point G (in the case of Mach-type reflection in the jet) or at the point B (in the case of totally sporadic reflection) is negative and, as we move downstream along this surface, approaches zero due to the influence of the plane of symmetry OO_1, the contact surface GG_1 (or BB_1) would have to possess a segment along which it is concave. With supersonic streamlining of the concave contact surface there would then occur an increase in pressure, causing a slowdown in the subsonic jet. But this is impossible due to the decrease in the cross-sectional area of the jet and the mass conservation conditions in the subsonic region of the flow. For this problem to be solvable in an approximation to an inviscous gas, it is necessary that the rarefaction wave be incident externally to the contact surface GG_1 in the region where it is concave. This would lead to a drop in pressure on the surface that is concave for the supersonic flow as well. It is precisely this circumstance that is observed in the case of Mach interaction of shock waves in a re-expanded jet [1]. If the flow is constructed in such a way that there is no such rarefaction wave, as is the case in the problems we are considering, then, in the formulation of the problem which assumes an inviscous gas, the problem of sporadic reflection in a jet will not have a solution. In order to state the closed formulation of the problem in this case it is necessary to take into account the ejective influence of the supersonic flow contiguous to the subsonic jet. In other words, in this case calculation of the interaction between the supersonic flow and a cocurrent subsonic jet must be carried out taking into account the viscosity of the gas or, more specifically, taking into account turbulent mixing in the mixing zone. A method of solving these types of problems was developed in [2, 3]. Within the framework of this method the stream in the subsonic jet is described in a boundary layer approximation, i.e., it is assumed that the transverse component of the pressure gradient $\partial p/\partial y$ is equal to zero and that the influence of the longitudinal derivatives of the components of the viscous stress tensor may be ignored. In fact, as is shown by the calculation of the parameters behind the shock wave OF for the schemes of flow considered over a broad range of variation of the governing parameters, the ratio of the pressure behind the shock wave OF at the point F to the pressure at the point O (p_F/p_O) is close to unity. For example, in the case of flow in the scheme of part C with $M_j = 3$ and $M_\infty = 4$, 5, 6, and 7, $p_F/p_O = 0.93$, 0.96, 0.98, and 0.99, respectively. This gives us a basis for setting $\partial p/\partial y = 0$ and for replacing the slightly curved shock wave OF by a forward shock. As for the longitudinal derivatives of the tangential stress tensor, their influence will apparently be substantial only in a small neighborhood of the point B or the point G. The boundary layer equations describing the stream in a subsonic jet and the equations describing the stream in a nonviscous supersonic flow are solved by means of marching (i.e., relative to the longitudinal coordinate) finite-difference methods (Crank–Nicholson [4] or MacCormack [5] type

approaches). The characteristics of an inviscous flow in an initial section (which coincides with the section connected to the Mach leg FO) are assumed to be known and are determined on the basis of the conditions under which the incident shock wave DA is reflected by the plane of symmetry OO_1 and the boundary of the jet upstream of the triple point F. The initial data in the subsonic jet are determined from the condition behind the forward shock OF. The solutions of the inviscous flow and boundary layer equations are combined by means of the generalized equations of viscous–inviscous interaction that were obtained in [2, 3]. These equations are a consequence of the threshold nature of the boundary conditions for the transverse component of the velocity v, which may be determined from the first-order equation, i.e., the continuity equation. They constitute a system of ordinary differential equations relative to the pressure in the jet $p_e(x)$ and the function $y^*(x)$ determining the shape of an effective displacement body streamlined by an external inviscous flow:

$$\Delta \frac{1}{\gamma p_e} \frac{dp_e}{dx} + q = A; \quad (2.1)$$

$$\frac{dy^*}{dx} = q; \quad (2.2)$$

$$q \frac{dp_e}{dx} - \rho_e u_e^2 \frac{dq}{dx} = \left. \frac{\partial p}{\partial x} \right|_{y=y^*(x)} (1 + q^2). \quad (2.3)$$

Here

$$\Delta = \int_0^\delta \left(1 - \frac{a^2}{u^2}\right) dy - \int_{y^*}^\delta \left(1 - \frac{a_e^2}{u_e^2}\right) dy;$$

$$A = \frac{\gamma - 1}{\gamma p_e} \int_0^\delta \frac{1}{u^2} \left(u \frac{\partial Q}{\partial y} - h \frac{\partial \tau}{\partial y}\right) dy;$$

$$\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y} \left(\frac{\rho \nu_t}{\mathrm{Pr}_t} \frac{\partial h}{\partial y}\right) + \rho \nu_t \left(\frac{\partial u}{\partial y}\right)^2; \quad \tau = \rho \nu_t \frac{\partial u}{\partial y};$$

ν_t is turbulent viscosity; $\delta(x)$, provisional (asymptotic) boundary of the viscous region; the subscript "e" denotes the values of the parameters in the inviscous flow on the surface of the displacement body; the remaining notation is well known. Equation (2.1) is a consequence of the continuity equation for the flow in a subsonic jet, while Eq. (2.3) is a consequence of the boundary condition of stagnation for an inviscous flow on the surface of the effective body [2, 3]. The coefficients Δ and A are determined by solving the boundary layer equations, while $\partial p/\partial y$ in (2.3) are determined by solving the Euler equations for an external inviscous flow. In a subsonic jet we have $\Delta < 0$ in the initial section. As the current in the jet subsequently accelerates due to the ejective influence of the external flow, Δ increases and becomes positive, passing through zero in the section $x = x_*$. In order for dp_e/dx to be bounded in this section, the condition A = q must also hold, i.e., we are dealing with an ordinary saddle-point singularity inherent to many problems related to acceleration of a subsonic flow (for example, currents in a near wake [6]).

A Cauchy problem may be stated for Eqs. (2.1)-(2.3). For this purpose it is necessary to specify the initial values of p_e, q, and y^* in the section x = 0 (section of the shock FO). The quantities p_e and $q = \tan \theta_F$ are determined from the Rankine–Hugoniot equations for conditions at the triple point F. As for $y^*(0)$, we have in the case of currents as given in part C, $y^*(0) = 1$, and in the case of currents in the scheme of part B, this quantity may be determined if the coordinate x of the point B or the point B' is found. The position of the forward shock FO in the case of totally sporadic reflection may be determined if the position of the point A is given. Thus, in all these types of currents it is necessary to determine the coordinate x of the point A or of B and B', which is found from the transit condition of a saddle-type singularity in the generalized equations of viscous–inviscous interaction (2.1)-(2.3) and is an eigenvalue of the problem. Once the distance x_A or x_B and $x_{B'}$ of the points A or B and B' is known, the position of the shock FO in the plane of flow may be determined for designated values of β_∞, M_∞, and M_j.

3. The results of calculations for Mach-type reflection in a jet and totally sporadic reflection are presented in Fig. 1b-1d. The calculations correspond to isoenergetic flow ($H = H_j = H$ – const, where H is total enthalpy) in the case of constant and equal values of the ratio of the specific heat capacities ($\gamma_\infty = \gamma_j = 1.4$) and with values of the governing

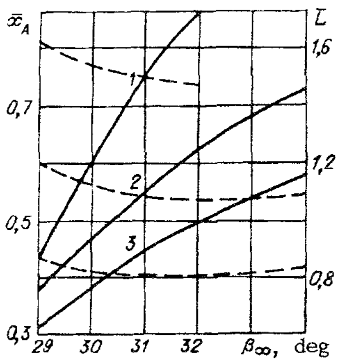

Fig. 2

parameters $M_\infty = 5$ and $M_j = 3$. We use the following algebraic model given by Prandtl [7] as our model of turbulent viscosity:

$$\nu_t = \varkappa / u_{max} - u_{min} / \delta, \quad Pr_t = 0{,}9.$$

The proportionality factor $\varkappa$ is set equal to 0.27 on the initial segment of the subsonic jet and to 0.32 on the main segment [7].

The patterns of flow depicted in Fig. 1b-1d correspond to distinct values of the slope β_∞ of the incident shock wave DA. If $\beta_\infty = 26°$, the scheme of flow with sporadic reflection in a jet of the re-reflected shock wave BF (Fig. 1b). With an increase in the slope of the shock wave DA to $\beta_\infty = 29°$ (Fig. 1c), there is sporadic reflection in the jet even in the case of the refracted wave AF. A further increase in the intensity of the incident shock wave DA leads to reconstruction of the flow and realization of the scheme of part C, i.e., totally sporadic reflection (Fig. 1d, $\beta_\infty = 29°$). Interestingly, if we proceed on the basis of the relation between the parameters of inviscous flow at the point F with the schemes of parts B and C, then, in some range of variation of the angle β_∞ both interaction schemes are possible for the same value of β_∞, i.e., in this sense the problem loses the property of uniqueness. Inasmuch as in an inviscous gas approximation the solution of the interaction problem in the presence of a strong shock wave FO does not, as noted earlier, exist, we may arrive at a final conclusion concerning the possibility of realizing both flow schemes with the same angle β_∞ once we have solved the problem taking into account the influence of viscosity. In certain cases, moreover, it is possible to extract a unique solution (with $\beta_\infty = 32°$ the solution corresponds to the scheme of Fig. 1d), while in other cases two solutions are obtained, for example, with $\beta_\infty = 29°$, corresponding to the scheme of Mach-type reflection in the jet (Fig. 1c), and the scheme of totally sporadic reflection (Fig. 1d). Apparently, either type of interaction may be realized depending on the history of the transition into this state at a value $\beta = \beta_\infty^*$ at which both types of interaction are possible. As β_∞ increases gradually, beginning with $\beta_{\infty cr}$, once the value $\beta = \beta_\infty^*$ is reached, flow with an internal Mach leg is achieved (Fig. 1c). As β_∞ decreases from values at which there is only totally sporadic reflection precisely this scheme of interaction will exist until the initial value $\beta = \beta_\infty^*$ is reached (Fig. 1d). Thus, a hysteresis phenomenon is observed. An analogous situation arises when a shock wave impinges on the near tail ([6], Chap. 6, Sec. 1).

Figure 2 illustrates the influence of the intensity of an incident shock wave on the geometric characteristics of a subsonic segment of the jet, in particular, the distance $\bar{x}_A = x_A/h_0$ (unbroken curves) and the length of the subsonic region $\bar{L} = L/h_0$ (broken curves) measured from the section of the strong shock wave FO to the section at which the mean Mach number is equal to unity. The calculations were conducted for the scheme of totally sporadic reflection with $M_j = 3$. Curves 1-3 correspond to $M_\infty = 5$, 6, and 7, respectively.

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NUMERICAL MODELING OF MACH REFLECTION FOR SOLITARY WAVES

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It is well known that when a surface solitary wave is incident on a vertical wall, located under an angle to the wave front, one can have either regular reflection, when only two waves are observed — the incident and the reflected, with their angles and amplitudes coinciding, or Mach reflection, when a ternary nonsymmetric configuration is generated. The studies [1-5] are devoted to investigating this phenomenon.

Perroud (see [3, 6]) has conducted experiments on wave reflection of dimensionless amplitude a = 0.08-0.38 in a wide range of incidence angles. Perroud observed that regular reflection is always realized for incidence angles $\psi_i \geqslant 45°$, while Mach reflection occurs for $\psi_i \leqslant 45°$. Quantitative characteristics of amplitudes and phases were also determined for wave reflection and Mach steps. According to Perroud's data, these parameters depend on the wave angle of incidence on the wall and are practically independent of its amplitude.

For low amplitude waves there exist theoretical results [2] on the resonance interaction of three solitary waves, providing, in particular, for $t \to \infty$ the asymptotic solution for Mach reflection problems. In that study it was obtained that the critical incidence angle ψ_i^*, distinguishing the two types of reflection, depends on the wave amplitude and equals $\sqrt{3a}$. The parameters of the ternary configuration also depend not only on the wave angle of incidence, but also on is amplitude. The predicted wave amplitude at the wall also differs substantially from experiment, reaching 4a in the Miles solution at the critical angle of incidence $\psi_i = \sqrt{3a}$.

In [4] this effect is modeled numerically with the use of approximate long-wave equations of low amplitude. The calculations were carried out for a = 0.05 and, on the whole, are in fair agreement with the Miles theory.

Specific experiments were carried out [3] so as to verify the theoretical model [2]. The reflection was treated of waves of amplitudes a = 0.1-0.15 for different angles. As a result of handling measurement data the author has expressed doubts concerning the validity of applying the Miles method to this problem. Thus, it must be recognized that a number of problems still remain open in this case.

In the present study we present results of a numerical investigation of a solitary wave reflection from a vertical wall for different amplitudes and angles of incidence. As mathematical models we use two discrete models of an incompressible fluid. The study substantially augments and refines the preliminary results, published in [7], of calculations for this problem, where a cruder grid was used and a solution was obtained for relatively small values of physical time.

1. Three-Dimensional Discrete Model. The given model is a generalization of the discrete model [8] to the three-dimensional case. The three-dimensional problem is considered of the interaction of a solitary wave over an even bottom with a rigid vertical wall, placed at an angle to the front. The z axis is directed upwards, and z = 0 corresponds to a flat unperturbed free surface. In the region Ω occupied by the fluid one introduces the regular grid

$$\Omega_h = \{\mathbf{r}'_\alpha = (x_\alpha, y_\alpha, z_\alpha) \,|\, \alpha = (i, j, k),\ i = 1, \ldots, M,\ j = 1, \ldots, N,\ k = 1, \ldots, L\},$$

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